A company is producing three different types of glue A, B and C. For each Kg of glue producted, the company has different costs in salary, electricity, and packaging (euro per kg), as shown in table 1.

TABLE 1

|  |  |  |  |
| --- | --- | --- | --- |
|  | Workers salary | Electricity | Packaging |
| Glue A | 2 | 5 | 1 |
| Glue B | 5 | 2 | 3 |
| Glue C | 8 | 3 | 2 |

We are interested in finding the amount of glue A, B and C, respectively

**Case 1**

The company wants to spend in one day of production exactly

|  |  |  |
| --- | --- | --- |
| Workers salary | Electricity | Packaging |
| 335 | 170 | 115 |

This problem can be formulated in a linear combination

that can be written also as a linear system

Firstly,, we compute the corresponding augmented matrix to check whether the system is consistent:

Secondly, we reduce the augmented matrix to echelon form:

Considering that the last column is not a pivot column, we can state that the system is consistent.

Therefore, the solution is =10, =15, =30

In conclusion, the amount of glue A, B and C produced per day are 10, 15, 30 kg respectively.

**Case 2**

The company wants to spend 620 euro per day, regardless of its subdivision between workers salary, electricity, and packaging.

Assuming that represent the costs for workers, elettricity, and packaging, we can write the following system:

Take Z as vector of solutions of the system: Z=

* Number of unknowns n = 6

AZ=b

=

According to the first case, 20; then at least there is 1 solution.

A=

Echelon form of A :

* Non-zero rows are 4 => Rank(A) = 4

A’ = A picture containing table

Description automatically generated

* Non-zero rows are 4 => Rank(A’) = 4
* R(A)=R(A’)=4<n=6 => infinite solutions

Considering that the system has 4 equations and 6 unknowns, we can affirm that there will be at least one free variable. Hence, with the number of variables exceeds the number of equations, the homogeneous system is consistent and has infinitely many solutions.

(reduced echelon form)